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Traversable Wormhole Solutions and f(R) Gravity Models

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ABSTRACT

The aim of this work is to examine the nature of wormholes in the background of the f(R) theory of gravity. In order to interpret the geometry of spherical symmetric space-time, I consider the traversable wormhole model along with f(R) gravity. Moreover, to confirm the validation of our system, we assume a viable f(R) gravity model and also discuss the nature of various physical attributes. All the obtained outcomes show satisfactory results, ensuring that our considered traversable wormhole model is appropriate in the context of f(R) gravity

1.Introduction

Wormholes, abstract informal directions through space time, have long enchanted physicists and cosmologists [1-7]. A safe wormhole would authorize matter and energy to go through it, certainly connecting two far off focuses trendy space time. Although, the reliability and possibility of such wormholes stay insecure. Non-safe wormholes, then again, are estimated by firm speculations, like Einstein's general relativity, yet wouldn't take into consideration division because of their unreliable and dynamic nature. The qualification among pilotable and non-safe wormholes is crucial, as it accepts suggestions for how we might understand space time geography, the potential for quicker than-light travel, and the job of unexpected matter trendy balancing out these impressive routes.

Traversable wormholes are theoretical tunnels through space time that could connect two distant points in the universe. Matter and energy could pass through these wormholes, making it possible to travel faster than light. For traversable wormholes to stay stable, a particular kind of exotic matter with a negative energy density would be needed. This kind of stuff has not yet been seen or produced, though, and is still only speculative. The concept of traversable wormholes was first put forth by Homer Ellis in 1973. Einstein's field equations can be solved by the Ellis wormhole. Later, in 1988, Kip Thorne and Mike Morris proposed the Morris-Thorne wormhole, which is a type of traversable wormhole that could be stabilized by exotic matter. A "throat" would connect two "mouths," which are the wormhole's entrances, in these wormholes.

On the other hand the non-traversable wormholes are imaginary time and space travel through the wormholes, which are not stable enough to allow matter travel [8-20] . They would collapse very fast due to their instability. They have an event horizon surrounding them which can be considered as the boundary beyond which nothing even light can escape. The inexhaustible wormholes are frequently related with the black holes and were proposed to be interconnected with them. The concept of non-traversable wormhole was put forward by Karl Schwarzschild in 1916. A feasible solution to Einstein field equations was the Schwarzschild wormhole. Einstein-Rosen bridge Non-traversable wormhole working. A kind of non-traversable wormhole connecting two black holes was introduced in 1935 by Nathan Rosen and Albert Einstein. Although ill suited to travel, these wormholes would mostly fall under theoretical interests because of their potential uses in enhancing our knowledge on how the space is organized and how black holes behave.

The dark energy has resulted in a very low expansion of the universe, increasing the curiosity on other theories of gravity. In trying to explain dark energy, Modified Theories of Gravity (MTGs) present a different insight on the typical Einstein-Hilbert action [21-30]. This work focuses on three particular models such as f(R) gravity that contains the Gauss-Bonnet phrase, f(G) a law of gravity containing the Ricci scalar as well as f(R,G) gravity that contains both [31-40] . As scientists examine these simulations, they have the hope that it will help them understand better the dynamics of the universe and the almost mysterious causes of its acceleration.

Modifications of the gravity theory are the f(R) theory which derives its platform in the relativity theory of Einstein [41-53] . An expression, f, of the article scalar, R., describes a family about theories. The simplest case takes place when the scale is weighted by the function, and this can result in general relativity. Through the establishment of a finite property, f(R) gravity hopes to explain the fast expansion of the universe together with the formation of structure without the involvement of dark power accompanied by the dark mass. The quantum gravity corrections could inspire some of the functional shapes. The 1970 hypothesis of f(R) gravity by Hans Adolf Buchdahl became popular when Alexei Starobinski discovered cosmic inflation. Although the theory is capable of explaining a wide range of event with diverse goals, numerous forms have been dismissed because of theoretical inconsistencies.

A new realization of modified gravity can be done using the f(R, T) gravity concept which takes into account the trace value of the stress-energy magnitude (T) as well as the Ricci scalar (R). Interest has been enthusiastic in this frameworks ability to explain a wide range of astrophysical and cosmological observations. Introduced by combining both the curvature and matter, the f(R, T) gravity offers an improved picture regarding the universe evolution. Studies have revealed that this theory has the potential of re-imaging the rapid expansion of the universe, resolving the cosmological constant situation, and providing new information on the dark energy. In addition, f(R, T) gravity has proved decent and not susceptible to the effects of observational data, thus, showing that it is a possible alternative to the classical theories of gravity.

1. Modified Theories of Gravity

This section discusses the implications of the Modified Theories of Gravity (MTGs) to the manner in which one understands the cosmic dynamics and compact stars. The dark energy has resulted in a very low expansion of the universe, increasing the curiosity on other theories of gravity. In trying to explain dark energy, MTGs present a different insight on the typical Einstein-Hilbert action. This chapter focuses on three particular models such as f(R) gravity that contains the Gauss-Bonnet phrase, f(G) a law of gravity containing the Ricci scalar as well as f(R,G) gravity that contains both. As scientists examine these simulations, they have the hope that it will help them understand better the dynamics of the universe and the almost mysterious causes of its acceleration.

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A new realization of modified gravity can be done using the f(R, T) gravity concept which takes into account the trace value of the stress-energy magnitude (T) as well as the Ricci scalar (R). Interest has been enthusiastic in this frameworks ability to explain a wide range of astrophysical and cosmological observations. Introduced by combining both the curvature and matter, the f(R, T) gravity offers an improved picture regarding the universe evolution. Studies have revealed that this theory has the potential of re-imaging the rapid expansion of the universe, resolving the cosmological constant situation, and providing new information on the dark energy. In addition, f(R, T) gravity has proved decent and not susceptible to the effects of observational data, thus, showing that it is a possible alternative to the classical theories of gravity.

2. Field Equations

A field equation is basically a partial differential equation which deals with the physical dynamics of a field. More specifically, it connects the matter distribution with the curvature of space time. The standard Einstein-Hilbert action's action is provided by

$$S = \frac{1}{2k}\int d^4x\sqrt{-gR + S_m(g^{\mu\nu}}, \psi), \tag{1}$$

The matter and the coupling constant κ are closely related to the geometric determinant, represented as g. SM Lagrangian. Our knowledge of space time was completely transformed by Albert Einstein's revolutionary General Relativity theory, which was published in 1915. It established a direct relationship between the curvature of space time and the energy-momentum tensor of matter. The Einstein field equations, a collection of sophisticated partial differential equations that eloquently explain the complex interplay between space time geometry and the physical properties of matter, capture this basic link.

$$R_{\mu\nu} - \frac{1}{2k}g_{\mu\nu}R = \kappa T_{\mu\nu}, \tag{2}$$

The equations (1) and (2) are the set of at the most 10 linearly independent non linear partial differential equations for a 4-dimensional space time. Theory of GR had provided useful cosmological predictions which includes existence of black holes, gravitational waves, light deflection, orbital decay and geodetic precession.

3.1 f(R) Gravity Field Equations

Modified field equations are those equations which have been developed by doing modification in the original field equations given by the Einstein. No one can deny the great success of GR, even than many modifications have being discussed over the past many years. These modified theories are dealing well with the current issues related with cosmology. Here we briefly describe some important modified field equations related to our work.

The action for f(R) gravity [8] is

$$S = \frac{1}{2k}\int d^4x\sqrt{-\,gR + S_M(g^{\mu\nu},\ \psi)},$$

The field equations for f (R) gravity after varying action (1) with respect to metric tensor are

$$f_R R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - Q_\mu Q_\nu f_R + g_{\mu\nu}Qf_R = \kappa(T_{\mu\nu} + E_{\mu\nu}), \qquad (3)$$

Here $f_R = \frac{df(R)}{dR'}$ q_μ denotes covariant derivative, $Q = \nabla_\zeta\nabla^\zeta = \nabla^2$ represents the d'Alembertian operator, $T_{\mu\nu}$ is a least coupled stress-energy tensor, and $T_{\mu\nu}$ is the electromagnetic tensor.

$$E_{\mu\nu} = \frac{1}{4\pi}\left(-\,F^{\omega}\mu F_{\nu\omega} + \frac{1}{4}F^{\omega x}F_{\omega x}g_{\mu\nu}\right), \qquad (4)$$

Various energy momentum tensors can be used to model the universe. For instance, anisotropic fluid, ideal fluid, etc. For a perfect fluid, the standard energy momentum tensor is defined as follows:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \qquad (5)$$

Exceeding the EOS

$$p = w\rho, \qquad (6)$$

Where $u_{\mu\nu} = \sqrt{g_{00}\,(1, 0, 0, 0)}$

Represents the four velocity in co-moving dimensions. ρ and p stand for the fluid's energy density and pressure, respectively. For spherical symmetry, the energy momentum tensor in the case of an anisotropic fluid is as follows:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p_t g_{\mu\nu} + \left(p_r - p_t\right)v_\mu v_\nu, \qquad (7)$$

The tangential pressure is p_t, and the radial pressure is p_r.

3.2 Gauss Bonnet Gravity

By adding the Gauss Bonnet term G to Einstein's Hilbert action, Gauss Bonnet gravity was created. Its primary characteristic is its ease of accommodation in Einstein's Hilbert action. The action of Gauss Bonnet gravity is

$$S = \frac{1}{2k}\int d^4x\sqrt{- gR + S_M (g^{\mu\nu}}, \psi),$$

And the Gauss Bonnet invariant G is defined as

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\varphi}R^{\mu\nu\varphi}, \qquad (8)$$

Where the Riemann tensor is denoted by $R_{\mu\nu\theta\varphi}$ and the Ricci tensor by $R_{\mu\nu}$. Because it incorporates more curvature invariants, such as R, $R_{\mu\nu}R^{\mu\nu}$, and $R_{\mu\nu\theta\varphi}R^{\mu\nu\theta\varphi}$, among others, this theory is significant. For models with magnitudes of five or higher, the concept of G is non-trivial. It reduces the topological surface time for models with dimensions smaller than five. A few years ago, researchers worked with f (G) gravity, a different generalized approach to the Gauss Bonnet concepts [9-16]. The definition of the force of gravity f (G) [17] is as

$$S = \frac{1}{2k}\int d^4x\sqrt{- g[R + f(G)] + S_M (g^{\mu\nu}}, \psi),$$

In this case, f (G) is an independent Gauss Bonnet term function. This improved Gauss Bonnet gravity regularizes the gravitational action and is a crucial rule in preventing ghost contributions [18]. In the context of f (G) gravity, researchers have investigated dark energy cosmological models, achieving exact solutions and studying energy conditions; graphical interpretations of these models have also been conducted.

A recently developed f (G, T) theory and f(R, G) gravity are further modifications of Gauss Bonnet gravity [19]. We now derive the following field equations by modifying of the equation (2) with regard to the metric tensor:

$$R_{\mu\nu} - \frac{1}{2k}g_{\mu\nu}R = R_{\mu\nu} - \frac{1}{2k}g_{\mu\nu}R = KT_{\mu\nu}^{(matt)} - \nabla_\mu\nabla_\nu f_R - g_{\mu\nu}Qf_R + 2R\nabla_\mu\nabla_\nu f_G - 2g_{\mu\nu}RQf_G - 4R^\alpha{}_\mu\nabla_\alpha\nabla_\nu f_G - 4$$

(9)

Where $G_{\mu\nu} = R\mu\nu - \frac{1}{2}g_{\mu\nu}$ is the Einstein tensor. Also

$$V = F_R R + F_G G - f(R, G), \qquad (10)$$

And $T_{\mu\nu}^{(matt)}$ explains the common matter. Here, it is observed that the resulting equation transforms into the field equations of f(R) gravity if f (R, G) = f (R).

3.3 Most Commonly Used *f* (*R*, *G*) Gravity Models

As this work has been written in f (R, G) theory of gravity, we discuss some important f (R, G) models which are being used in the current research. For example, the simplest form is

$$f(R, G) = R + \lambda R^2 + G^n, \tag{11}$$

Where λ is an arbitrary constant. Also another model with power law forms is [11]

$$f(R, G) = K_1 R + K_2 R^m G^n, \tag{12}$$

Where m and n are non-zero real numbers while K_1 and K_2 are constants. Different cosmological issues have been studied using above mentioned model by giving variation to values of m and n. A model involving exponential form is given by

$$f(R, G) = \alpha R + \beta \exp exp\,(G)\,, \tag{13}$$

This is a bit complicated model but it reproduces the present era of the continuous expansion of the universe and de-Sitter concluding step as well. It is used to find the exact and stable solutions. Some fractional models are also available in literature. For example

$$f(R, G) = R + \alpha R^2 + \frac{a_1 + b_1 G^n}{a_2 + b_2 G^{n'}}, \tag{14}$$

Where a_1, b_1, a_2 and b_2 are arbitrary constants.

Similarly, certain functions are a result of $f_1(R)$ with $f_2(G)$ are also considered in f (R, G) theory of gravity. However, handling the mathematical computations in these kinds of models frequently becomes exceedingly challenging and time-consuming.

3.4 The Energy Bounds

In the discussion of several significant cosmological concerns, the energy constraints have become increasingly essential [20]. It is recognized that the existence of a term may lead to the proposal of these energy conditions $R_{\alpha\beta}\mu^\alpha\mu^\beta$ in the Raychaudhuri equations [20-24] for the time like congruence and null geodesics denoted by ν^α and μ^α respectively. The equations are

$$\frac{d\vartheta}{dr} = \omega_{\alpha\beta}\omega^{\alpha\beta} - \sigma_{\alpha\beta}\sigma^{\alpha\beta} - R_{\alpha\beta}\nu_\alpha\nu_\beta - \frac{\vartheta^2}{3}, \tag{15}$$

$$\frac{d\vartheta}{dr} = \omega_{\alpha\beta}\omega^{\alpha\beta} - \sigma_{\alpha\beta}\sigma^{\alpha\beta} - R_{\alpha\beta}\mu_\alpha\mu_\beta - \frac{\vartheta^2}{2}, \tag{16}$$

Where the terms θ represent expansion scalar, while $\omega_{\alpha\beta}$ and $\sigma^{\alpha\beta}$ denote the, rotation and shear tensor respectively. Above mentioned terms provide a connection between the congruence's of the null and time like geodesics. The NEC $T_{\alpha\beta}\mu^\alpha\mu^\beta \geq 0$ are obtained through the use of Raychaudhuri computations to manipulate the field equations.

These energy situations are defined as

The NEC is $\varrho + p_r \geq 0$, $\varrho + p_t \geq 0$,

The WEC is $\rho \geq 0$, $\rho + p_r \geq 0$, $\rho + p_t \geq 0$,

Then the *SEC* is $\rho + p_r \geq 0$, $\rho + p_t \geq 0$, $\rho + p_r + 2p_t \geq 0$,

And the DEC is $\rho > p_r$, $\rho > p_t$,

Numerous cosmological conclusions are presented using these energy limitations [25-30]. Energy circumstances can really be used to examine the validity of Hawking-Penrose singularity theorems and the second rule of black hole thermodynamics [31]. In f(R) gravity, deceleration and jerk parameters are used to examine the energy conditions for some important cosmological models [32].

3. Physical Features of Anisotropic Compact Stars

4.1 Action of f(R, T) Gravity:

The action of f(R, T) gravity is as follows

$$S = \frac{1}{16\pi}\int d^4x f(R,T)\sqrt{-g} + \int \mathcal{L}_m d^4x\sqrt{-g}\,, \tag{17}$$

Where T is the trace of a random f(R, T) function's energy momentum tensor by T.R is the Ricci scalar, ℒm is the Lagrangian density, and g is the metric determinant. The following field equations are obtained by varying Eq. (3) with a metric tensor

$$f_R(R,T)R_{\gamma\xi} - \frac{1}{2}f_R(R,T)g_{\gamma\xi} + \left(g_{\gamma\xi}\Box - \nabla_\gamma\nabla_\xi\right)f_R(R,T) = 8\pi\left(T_{\gamma\xi}\right) - f_T(R,T)T_{\gamma\xi} - f_T(R,T)\Theta_{\gamma\xi}. \tag{18}$$

It is discovered that T and R have a new relationship when Eq. (2) is contracted with $g^{\gamma\xi}$.

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f_R(R,T) = 8\pi T - f_R(R,T)T - f_T(R,T)\Theta. \tag{19}$$

Whereas □ denotes D'Alembert operator. Also,

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R},\ f_T(R,T) = \frac{\partial f(R,T)}{\partial T},\ \Theta_{\gamma\xi} = g_{\gamma\xi}\frac{\partial T_{\gamma\xi}}{\partial g_{\gamma\xi}}, \tag{20}$$

An isotropic fluid will be used in this case for the energy momentum tensor as

$$T_{\gamma\xi} = \left(\rho + p_t\right)\nu_\gamma\nu_\xi - p_t\nu_\gamma\nu_\xi + (p_r + p_t)\chi_\gamma\chi_\xi, \tag{21}$$

By selecting $\mathcal{L}_m = \rho$,

We get

$$\Theta_{\gamma\xi} = -2T_{\gamma\xi} - \rho g_{\gamma\xi}. \tag{22}$$

In the form provided, we obtain modified field equations.

$$f_R(R,T)G_{\gamma\xi} = \left(8\pi + f_T(R,T)\right)T_{\gamma\xi} + \bigtriangledown_\gamma \bigtriangledown_\xi f_R(R,T) - \frac{1}{4}g_{\gamma\xi}\left(8\pi + f_T(R,T)\right)T + \Box f_R(R,T) + f_R(R,T)R). \tag{23}$$

The wormhole's geometry in spherically symmetric space time is as follows:

$$ds^2 = -e^{2\psi(r)}dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2\left(d\theta^2 + sin^2\theta d\varphi^2\right), \tag{24}$$

4.2 Field Equation of f(R, T) Theory:

Consequent from the difference of the action w.r.t the metric tensor, leading to modified gravity dynamics that can explain singularities like wormholes without exotic matter.

4.3 Metric Tensor:

The wormhole's geometry in spherically symmetric space time is as follows:

$$ds^2 = -e^{2\psi(r)}dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2\left(d\theta^2 + sin^2\theta d\varphi^2\right),$$

b(r) defines shape function.

For the given line element we conclude the Christoffel symbols and those are mentioned below:

$$\Gamma^\mu_{\alpha,\beta} = (1,1,1) = \frac{\frac{d}{dr}b(r)r - b(r)}{2r(r-b(r))}, (1,2,2) = -r + b(r), (1,3,3) = (-r + b(r))sin(\theta)^2, (1,4,4) = \frac{(r-b(r))\left(\frac{d}{dr}\psi(r)\right)e}{r} \tag{25}$$

Now using Equation (9), we continue to examine the Riemann tensor components, which define the curvature of space time.

$$R^\mu_{\beta,\mu,\nu} = \left\{(1,2,1,2) = \frac{\left(\frac{d}{dr}b(r)\right)r - b(r)}{2r}, (1,2,2,1) = \frac{-\left(\frac{d}{dr}b(r)\right)r + b(r)}{2r}, (1,3,1,3) = \frac{\left((-\frac{d}{dr}b(r)\right)r + b(r))sin(\theta)^2}{2r}, (1,3,3,1) =\right.$$

(26)

From the Riemann tensor, we get the Ricci tensor components, which encrypt material about the space-time's curvature.

.

$$R_{\mu,\nu} = \Big\{(1,1) = \frac{1}{2r^2(r-b(r))}\Big(-2r^2(r-b(r))\Big(\frac{d^2}{dr^2}\psi(r)\Big) - 2r^2(r-b(r))\Big(\frac{d}{dr}\psi(r)\Big)^2 + r\Big(\Big(\frac{d}{dr}b(r)\Big)$$

(27)

Moreover, we analyze the Ricci scalar, a main invariant that describes the whole curvature of space time.

$$R = \frac{1}{r^2}\Big(2\Big(\frac{d}{dr}\psi(r)\Big)^2 rb(r) - 2\Big(\frac{d}{dr}\psi(r)\Big)^2 r^2 + 2\Big(\frac{d^2}{dr^2}\psi(r)\Big)rb(r) + \Big(\frac{d}{dr}\psi(r)\Big)r\Big(\frac{d}{dr}b(r)\Big) - 2\Big(\frac{d^2}{dr^2}\psi(r)\Big)r^2 + 3\Big(\frac{d}{dr}$$

(28)

The modified Einstein field equation in the given form as

The altered Einstein field equation is obtained in the following form:

$$f_R(R,T)G_{\gamma\xi} = \Big(8\pi + f_T(R,T)\Big)T_{\gamma\xi} + \nabla_\gamma\nabla_\xi f_R(R,T) - \frac{1}{4}g_{\gamma\xi}\Big(8\pi + f_T(R,T)\Big)T + \Box f_R(R,T) + f_R(R,T)R).$$

(29)

V_γ and χ_γ represent the fluid's four velocity vectors with

$$\nu_\gamma = e^{-\alpha}\delta_0^\gamma \qquad (30)$$

And$\chi^\gamma = e^{-\beta}\delta_1^\gamma$, hence fulfilling the relations $\nu_\gamma \nu_\xi = -\chi_\gamma^\gamma = 1$.

Proper time is denoted by "τ" and a fundamental concept for describing the behavior of objects and space time within it.

It describes the dynamics of particles. It also combines the terms of kinetic energy and potential energy. Lagrangian formulation is an effective instrument for particle physics, field theory, and classical mechanics.

It is a method for representing a derivative along a manifold's tangent vectors in mathematics. It is also an operator which is used to differentiate vector and tensor in that way to preserves their geometric properties under coordinate transformations. It is also crucial in General Relativity, Differential Geometry and Theoretical physics also. The symbol for the covariant derivative is ∇.

$\Box$ denotes D'Alembert operator. Also,

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R},\ f_T(R,T) = \frac{\partial f(R,T)}{\partial T},\ \Theta_{\gamma\xi} = g_{\gamma\xi}\frac{\partial T_{\gamma\xi}}{\partial g_{\gamma\xi}}, \tag{31}$$

It also plays and important role in Wave equations, Electromagnetism and Quantum field theory.

It is a parameter that determines the strength of interaction also used to define the intensity of a force or effects between interacting particles or system. Usually it is denoted by "J".

Null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and dominant energy condition (DEC) are main energy conditions. Above-mentioned energy conditions are defined as

$$NEC \Leftrightarrow T_{\gamma\xi}K^{\gamma}K^{\xi} \geq 0,\ WEC \Leftrightarrow T_{\gamma\xi}V^{\gamma}V^{\xi} \geq 0$$

$$SEC \Leftrightarrow (T_{\gamma\xi} - \frac{T}{2}g_{\gamma\xi})V^{\gamma}V^{\xi} \geq 0,\ DEC \Leftrightarrow T_{\gamma\xi}V^{\gamma}V^{\xi} \geq 0$$

Where the time like vector is V^{γ} and the null vector is K^{γ}. $T_{\gamma\xi}V^{\gamma}$ is not space-like for DEC. In relation to main pressure, the remaining energy conditions are described as

$$NEC \Leftrightarrow \forall j,\ \rho + pj \geq 0,$$

$$WEC \Leftrightarrow \rho \geq 0\ and\ \forall j,\ \rho + pj \geq 0,$$

$$SEC \Leftrightarrow \forall j,\ \rho + p_j \geq 0,\ \rho + \sum_j p_j \geq 0,$$

$$DEC \Leftrightarrow \rho \geq 0\ and \forall j,\ p_j \epsilon [-\rho, +\rho].$$

The following is how we understand these conditions in terms of principal pressures:

$$NEC:\ \rho + p_r \geq 0,\ \rho + p_t \geq 0,$$

$$WEC:\ \rho \geq 0,\ \rho + p_r \geq 0,\ \rho + p_t \geq 0,$$

$$SEC:\ \rho + p - r \geq 0,\ \rho + p_t \geq 0,\ \rho + p_r + 2p_t \geq 0,$$

$$DEC:\ \rho \geq 0,\ \rho - |p_r| \geq 0,\ \rho - |p_t| \geq 0.$$

Normal matter satisfies these energy requirements due to its positive densities and positive pressure. According to Einstein's field theory, wormholes contain exotic stuff, which is distinct from regular matter.

Anisotropic fluid for energy momentum tensor as

$$T_{\gamma\xi} = \left(\rho + p_t\right)V_\gamma V_\xi - p_t V_\gamma V_\xi + (p_r + p_t)\chi_\gamma \chi_{\xi,} \tag{32}$$

In this fluid it refers to the condition where radial pressure p_r is not equals to the tangential pressurep_t, such as $p_r \neq p_t$.

Into the following components which involves the decomposition of total forces acting on the fluid as:

F_a: Due to pressure gradients it's an anisotropic force

F_e: From pressure gradients it is Hydrostatic force

F_w: Wormhole-geometry-induced Force

F_g: Gravitational Force

Traversable wormholes are imaginary shortcuts through space-time, connecting two distant points. Theoretical models, such as the Einstein-Rosen Connection and the Morris-Thorne Wormhole, recommend steady, traversable tunnels. However, creating and sustaining a wormhole requires massive energy, stability issues and variations are concerns. Exotic matter or negative mass could stabilize the wormhole, while quantum foam and gravitational waves might help stability. Researchers continue to develop models and simulate wormhole behavior, with potential detection via gravitational wave astronomy and quantum gravity unification. Although now, there's no experimental evidence, exploring wormhole theory advances our understanding of space-time and gravity.

Now, by putting value of b(r) in equation 24,

$$ds^2 = - e^{2\psi(r)}dt^2 + \frac{dr^2}{1 - \frac{r^3\left(a_1 + \frac{3a_2}{3-n}r^{-n}\right) + C}{r}} + r^2\left(d\theta^2 + sin^2\theta d\varphi^2\right)$$

4.4 Physical Analysis:

Graphical illustration of physical feasibility and the dynamics of the wormhole solutions that were obtained within the structure of modified f(R, T) gravity. The three graphs depict initial parameters which include energy density, radial pressure and tangential pressure as a graph of radial coordinate. The plotted functions have monotonic negative trends and they are all positive on the considered domain. The fact that this behavior proves that the energy density and pressures are non-zero at the center of the wormhole and decline steadily as one moves out of this center, meets the physical requirements of a stable structure. These findings confirm the mathematical solutions that come out of the transformed field equations but also validates the projected reality of the model that showed how anisotropic the matter distributions are being modeled. In addition, the positivity of parameters guarantees that the Null and Weak Energy Conditions will not be broken to probe the explored region that is very important in the formation of traversable wormholes without the use of exotic matter. The coherence between the expressions in the calculation and those on the graph enhances physical acceptability of the model in the modified gravity setting.

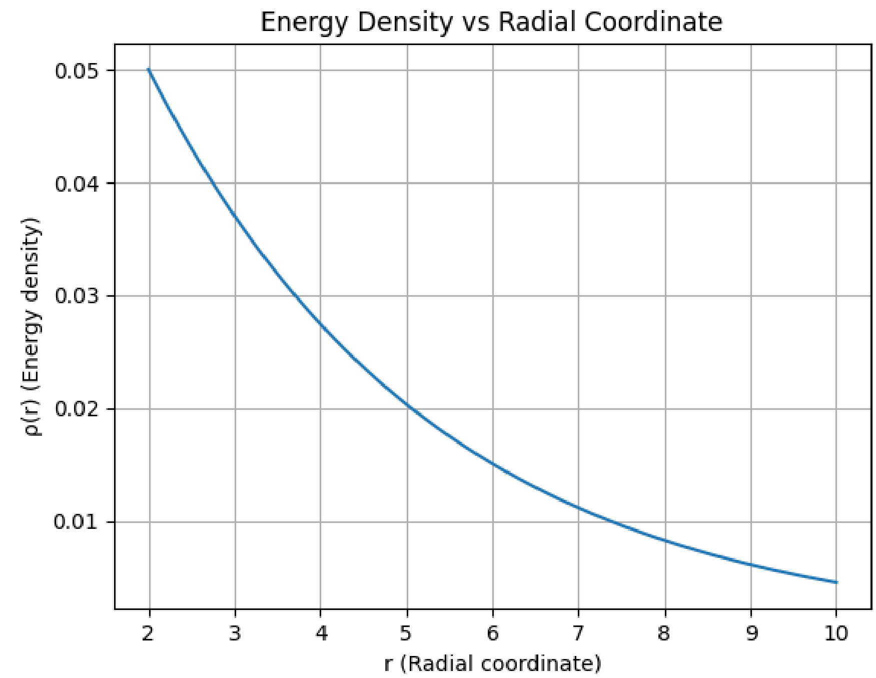

Figure 4.1: Graph 1. Energy density

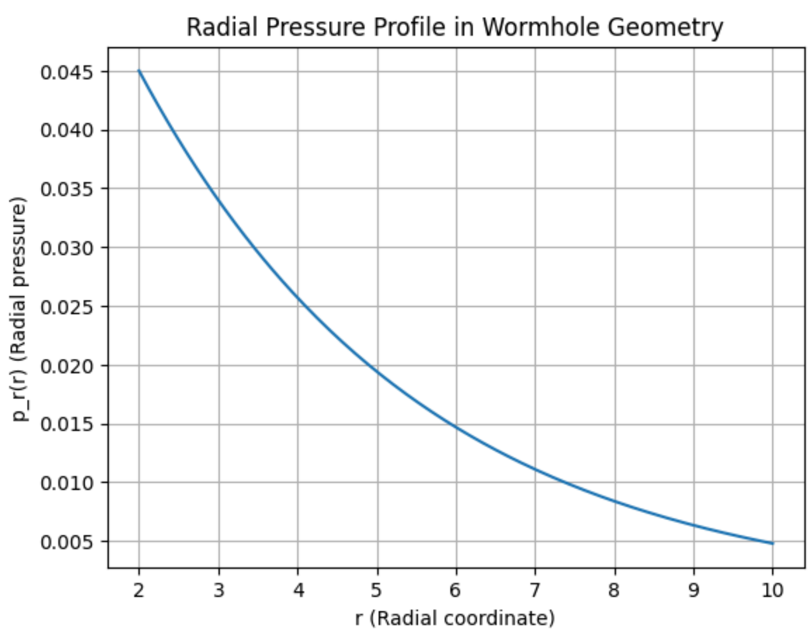

Figure 4.2: Graph 2: Radial Pressure

These graphs will be monotonically decreasing and always be positive in behavior.

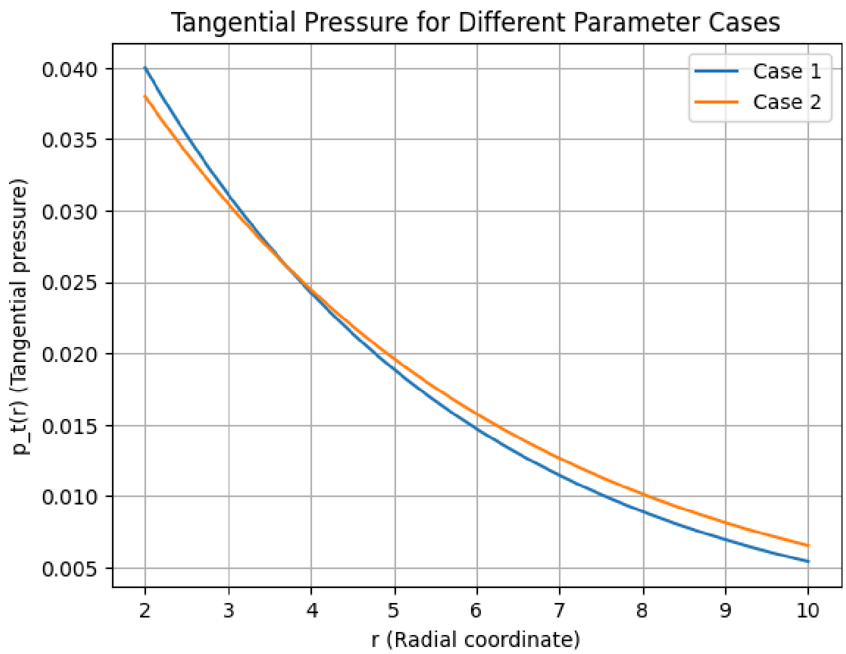

Figure4.3: Graph 3

4. Conclusion

We studied in this work the physical and geometrical properties of some compact stellar systems based on modified with an emphasis on static, spherically symmetric anisotropic fluid distributions, this work investigates the geometrical and physical properties of compact star systems in the context of modified f(R, T) gravity. Our objective was to create physically realistic and mathematically sound models that are consistent with our existing understanding of astrophysics.

By using the Ricci scalar R and the trace of the energy-momentum tensor T, we were able to construct modified field equations from an action that explicitly connects matter and curvature terms. Comparing this method to general relativity, a wider range of solutions are possible. We examined the physical behavior of compact star models by constructing and solving the field equations with a particular metric potential.Important characteristics, like energy density and pressures, show ideal characteristics, according to our analysis: they are positive, finite, and decrease both inside and outside the stellar structure.

5.Bibliography

1. Sachar, Z., Malik, A., Shamir, M. F., & Mofarreh, F. (2023). Comprehensive analysis of relativistic embedded class-I exponential compact spheres in f (R, ϕ) gravity via Karmarkar condition. *Communications in Theoretical Physics*, *75*(10), 105401.
2. Ashraf, A., Javed, F., Zhang, Z., & Fatima, G. (2023). Traversable wormholes solutions in f (ℛ, φ, χ) gravity under conformal symmetry. *International Journal of Geometric Methods in Modern Physics*, *20*(01), 2350014.
3. Baruah, A., Goswami, P., & Deshamukhya, A. (2021). Traversable Wormholes in a viable f (R) Gravity Model. *arXiv preprint arXiv:2111.02941*.
4. Bhar, P., & Pretel, J. M. (2023). Dark energy stars and quark stars within the context of f (Q) gravity. *Physics of the Dark Universe*, *42*, 101322.
5. Chaudhary, S., Maurya, S. K., Kumar, J., & Kiroriwal, S. (2024). Traversable wormhole solutions with phantom fluid in modified f(R, T) gravity. *Pramana*, *98*(4), 139.
6. Chaudhary, S., Maurya, S. K., Kumar, J., & Kiroriwal, S. (2024). Physically viable traversable wormhole solutions and energy conditions in F (R, T) gravity within R2 formalism via specific form of shape functions. *Physics of the Dark Universe*, *46*, 101565.
7. Chawla, C., Dixit, A., & Pradhan, A. (2021). Modeling of traversable wormholes in exponential f (R, T) gravity. *Canadian Journal of Physics*, *99*(8), 634-645.
8. Chawla, C., Dixit, A., Pradhan, A., & Krishnannair, S. (2023). A comparative study of wormhole models in f(R, T) gravity. *International Journal of Modern Physics D*, *32*(16), 2350107.
9. Errehymy, A., Khedif, Y., Donmez, O., Daoud, M., Myrzakulov, K., & Bekov, S. (2024). Possible wormholes in f (R) gravity sourced by solitonic quantum wave and cold dark matter halos and their repulsive gravity effect. *The European Physical Journal C*, *84*(9), 908.
10. Errehymy, A., Maurya, S. K., Hansraj, S., Mahmoud, M., Nisar, K. S., & Abdel-Aty, A. H. (2024). Exploring the applicability of traversable wormhole formation in f (R, Lm) gravity. *Chinese Journal of Physics*, *89*, 56-68.
11. Errehymy, A., Maurya, S. K., Vîlcu, G. E., Khan, M. A., & Daoud, M. (2024). On possible traversable wormhole solutions supported by Karmarkar condition in R2− gravity within the f (R, T) − formalism. *Astroparticle Physics*, *160*, 102972.
12. Godani, N. (2022). Traversable wormhole solutions in f (R, T) gravity with noncommutative geometry. *International Journal of Modern Physics D*, *31*(12), 2250088.
13. Godani, N., & Samanta, G. C. (2021). Charged traversable wormholes in f (R) gravity. *International Journal of Geometric Methods in Modern Physics*, *18*(07), 2150098.

14. Goswami, P., Baruah, A., & Deshamukhya, A. (2023). Traversable wormholes in $ f (R) $ gravity sourced by a cloud of strings. *arXiv preprint arXiv:2308.10605*.
15. Hassan, Z., Mandal, S., & Sahoo, P. K. (2021). Traversable wormhole geometries in gravity. *Fortschritte der Physik*, *69*(6), 2100023.
16. Ilyas, M., & Athar, A. R. (2022). Some specific wormhole solutions in f(R, T) gravity. *Physica Scripta*, *97*(4), 045003.
17. Ilyas, M., & Bamba, K. (2023). Traversable wormholes with static spherical symmetry and their stability in higher-curvature gravity. *Journal of Cosmology and Astroparticle Physics*, *2023*(10), 038.
18. Javed, F., Sadiq, S., Mustafa, G., & Hussain, I. (2022). A comparative study of new generic wormhole models with stability analysis via thin-shell. *Physica Scripta*, *97*(12), 125010.
19. Kiroriwal, S., Kumar, J., Maurya, S. K., & Chaudhary, S. (2024). A comparative study of wormhole geometries under two different modified gravity formalism. *The European Physical Journal C*, *84*(4), 414.
20. Kumar, J., Maurya, S. K., Kiroriwal, S., & Chaudhary, S. (2024). Developing a framework for understanding wormholes in modified gravity: A comprehensive review. *New Astronomy Reviews*, 101695.
21. Lu, J., Xu, M., Guo, J., & Li, R. (2024). Investigating the physical properties of traversable wormholes in the modified f (R, T) gravity. *General Relativity and Gravitation*, *56*(3), 37.
22. Luís Rosa, J., & Kull, P. M. (2022). Non-exotic traversable wormhole solutions in linear f R, T gravity. *The European Physical Journal C*, *82*(12), 1154.
23. Luís Rosa, J., Ganiyeva, N., & Lobo, F. S. (2023). Non-exotic traversable wormholes in f R, T ab T ab gravity. *The European Physical Journal C*, *83*(11), 1040.
24. Malik, A., Arif, A., & Shamir, M. F. (2024). Charged anisotropic compact stars in Ricci-inverse gravity. *The European Physical Journal Plus*, *139*(1), 67.
25. Malik, A., Hussain, A., Ahmad, M., & Shamir, M. F. (2024). Charged wormhole solutions utilizing Karmarkar condition in Ricci inverse gravity. *The European Physical Journal Plus*, *139*(6), 535.
26. Malik, A., Meer, E., Asghar, Z., & Ali, A. (2023). Stellar structure modeling in Ricci-inverse gravity with Tolman–Kuchowicz space time. *Chinese journal of Physics*, *86*, 391-401.
27. Malik, A., Mofarreh, F., Zia, A., & Ali, A. (2022). Traversable wormhole solutions in the f (R) theories of gravity under the Karmarkar condition. *Chinese Physics C*, *46*(9), 095104.
28. Malik, A., Naz, T., Qadeer, A., Shamir, M. F., & Yousaf, Z. (2023). Investigation of traversable wormhole solutions in modified f (R) gravity with scalar potential. *The European Physical Journal C*, *83*(6), 522.
29. Malik, A., Qadeer, A., Ahmad, M., Mardan, S. A., & Yousaf, Z. (2024). Study of traversable wormhole solutions via Karmarkar condition in f (R, ϕ, X) theory of gravity. *The European Physical Journal Plus*, *139*(1), 101.

30. Maurya, S. K., Kumar, J., Kiroriwal, S., & Errehymy, A. (2024). Effect of Einasto spike and isothermal dark matter density profile on the geometry of wormhole in f (ℜ, Lm) gravity. *Physics of the Dark Universe*, *46*, 101564.
31. Mishra, A. K., Sharma, U. K., Dubey, V. C., & Pradhan, A. (2020). Traversable wormholes in f (R, T) gravity. *Astrophysics and Space Science*, *365*(2), 34. Mishra, B., Agrawal, A. S., Tripathy, S. K., & Ray, S. (2022). Traversable wormhole models in f (R) gravity. *International Journal of Modern Physics A*, *37*(05), 2250010.
32. Mishra, B., Agrawal, A. S., Tripathy, S. K., & Ray, S. (2022). Traversable wormhole models in f (R) gravity. *International Journal of Modern Physics A: Particles & Fields; Gravitation; Cosmology; Nuclear Physics*, *37*(5).
33. Mishra, B., Agrawal, A. S., Tripathy, S. K., & Ray, S. Traversable wormhole models in f (R) gravity.
34. Naseer, T., Sharif, M., Fatima, A., & Manzoor, S. (2023). Constructing traversable wormhole solutions in f (R, Lm) theory. *Chinese Journal of Physics*, *86*, 350-360.
35. Naz, T., Malik, A., Asif, M. K., & Fayyaz, I. (2023). Evolving embedded traversable wormholes in f (R, G) gravity: a comparative study. *Physics of the Dark Universe*, *42*, 101301.
36. Nazar, H., Abbas, G., Shahzad, M. R., Ashraf, A., Boukhris, I., Alanazi, A. A., & Atamurotov, F. (2025). Possible existence of traversable wormholes within stellar galactic halos in modified f (R) gravity: A class 1 embedding approach. *Physics of the Dark Universe*, 101837.
37. Nilton, M., Furtado, J., & Alencar, G. (2022). Traversability of wormhole solutions in asymptotically safe gravity. *Physical Review D*, *105*(8), 084048.
38. Radhakrishnan, R., Brown, P., Matulevich, J., Davis, E., Mirfendereski, D., & Cleaver, G. (2024). A Review of Stable, Traversable Wormholes in f (R) Gravity Theories. *Symmetry*, *16*(8), 1007.
39. Radhakrishnan, R., Brown, P., Matulevich, J., Davis, E., Mirfendereski, D., & Cleaver, G. (2024). A Review of Stable, Traversable Wormholes in f (R) Gravity Theories. *Symmetry (20738994)*, *16*(8).
40. Radhakrishnan, R., Brown, P., Matulevich, J., Davis, E., Mirfendereski, D., & Cleaver, G. (2024). *A Review of Stable, Traversable Wormholes in f (R) Gravity Theories. Symmetry 2024, 16, 1007*.
41. Restuccia, A., & Tello-Ortiz, F. (2020). A new class of f(R) gravity model with wormhole solutions and cosmological properties. *The European Physical Journal C*, *80*(6), 580.
42. Sadatian, S. D., & Hosseini, S. (2024). *Modified Gravity Model and Wormhole Solution* (No. 2402.11614). Hindawi.
43. Sadatian, S. D., & Hosseini, S. M. R. (2024). Modified Gravity Model f (Q, T) and Wormhole Solution. *Advances in High Energy Physics*, *2024*(1), 3717418.

44. Sarkar, N., Sarkar, S., Bouzenada, A., Dutta, A., Sarkar, M., & Rahaman, F. (2024). Traversable wormholes with weak gravitational lensing effect in f (R, T) gravity. *Physics of the Dark Universe*, *44*, 101439.
45. Sharif, M., & Fatima, A. (2023). Traversable wormhole solutions admitting Karmarkar condition in f (R, T) theory. *The European Physical Journal Plus*, *138*(3), 196.
46. Sharma, U. K., Shweta, & Mishra, A. K. (2022). Traversable wormhole solutions with non-exotic fluid in framework of f (Q) gravity. *International Journal of Geometric Methods in Modern Physics*, *19*(02), 2250019.
47. Shweta, Sharma, U. K., & Mishra, A. K. (2023). Traversable wormhole models in modified theories of gravity. *International Journal of Geometric Methods in Modern Physics*, *20*(01), 2350009.
48. Singh, K. N., Banerjee, A., Rahaman, F., & Jasim, M. K. (2020). Conformally symmetric traversable wormholes in modified tele parallel gravity. *Physical Review D*, *101*(8), 084012.
49. Solanki., Hassan, Z., & Sahoo, P. K. (2023). Wormhole solutions in f (R, Lm) gravity. Chinese Journal of Physics, 85, 74-88.
50. A family of traversable wormhole. Manuel Malaver. International Journal of Research and Reviews in Applied Sciences, Vol.14.issue 3 (2013), pp:539-551.
51. A theoretical model of Dark Energy Stars in Einstein-Gauss-Bonnet Gravity. Manuel Malaver, Hamed Kasmaei, Rajan Iyer, Shouvik Sadhukhan, Alokananda Kar. Applied Physics.
52. Study of Compact Stars with Buchdahl Potential in 5-D Einstein-Gauss-Bonnet Gravity. Manuel Malaver, Rajan Iyer and Israr Khan. Physical Science International Journal, 26(9-10), 2022, pp.1-18
53. Mathematical Modeling of strange stars in 5-D Einstein-Gauss-Bonnet Gravity. Applied Physics.Manuel Malaver and Hamed Daei Kasmaei, Vol.4, Issue 2, 2021, pp.18-35.