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Influence of Transverse Roughness and Porous Medium on Conjugate Heat Transfer in an Inclined Nanofluid-Filled Cavity: A Lattice Boltzmann Study

Rakesh Manilal H. Patel
Department of Mathematics
Government Science College, Sector – 15
Gandhinagar, Gujarat State, India.

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Correspondence:

E-mail:rmpatel2711@gmail.com

ABSTRACT

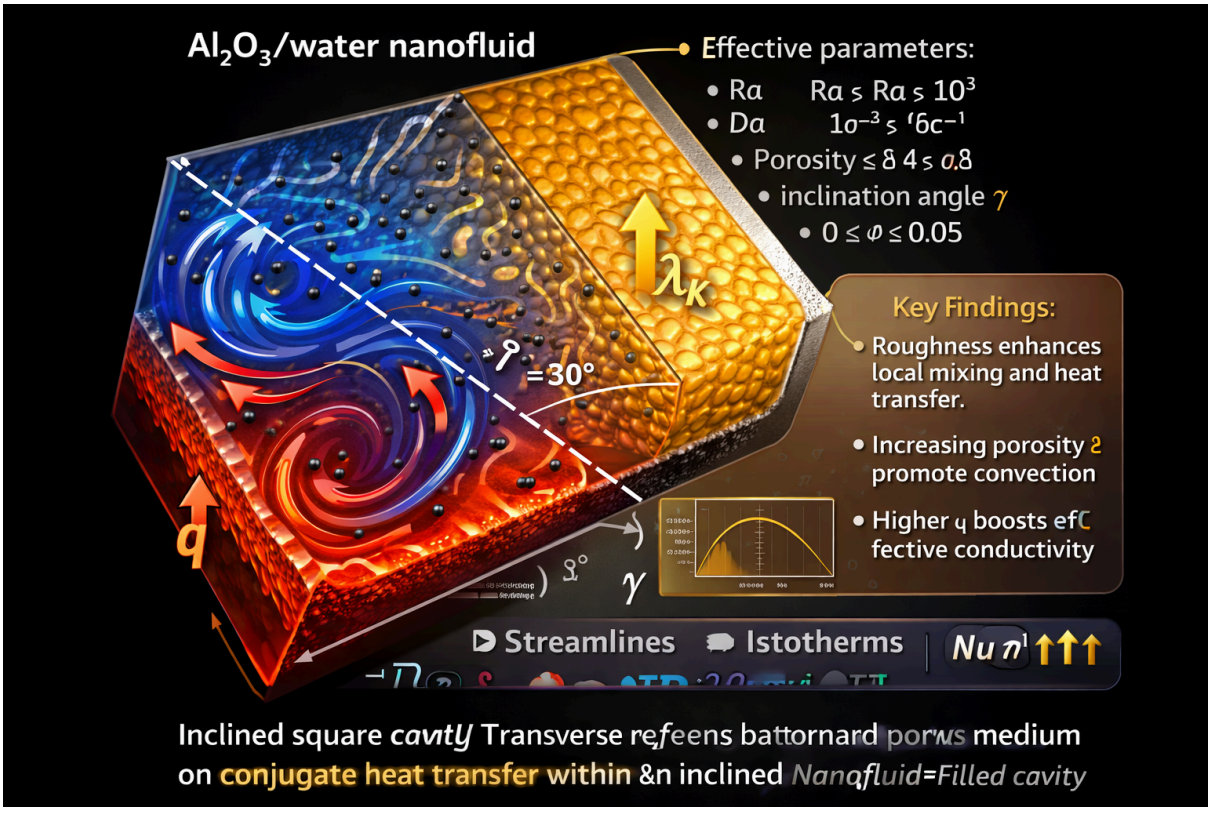
This study numerically investigates the influence of transverse wall roughness and porous medium characteristics on conjugate heat transfer within an inclined cavity filled with Al_2O_3/water nanofluid using the Lattice Boltzmann Method (LBM). The physical model consists of a two-dimensional inclined enclosure with roughened heated walls, where solid–fluid heat conduction coupling is considered to account for conjugate effects. The porous structure is modeled using the Darcy–Forchheimer formulation, while nanofluid thermophysical properties are evaluated based on effective medium theory. The governing equations for momentum and energy transport are solved using a double-distribution LBM scheme. The effects of key dimensionless parameters—including Rayleigh number ($10^3 \le Ra \le 10^6$), Darcy number ($10^{-5} \le Da \le 10^{-2}$), porosity ($0.4 \le \varepsilon \le 0.9$), roughness amplitude and pitch, nanoparticle volume fraction ($0 \le \varphi \le 0.05$), and cavity inclination angle ($0° \le \gamma \le 90°$)—are systematically examined. Results reveal that transverse roughness significantly enhances local mixing and disrupts thermal boundary layers, leading to noticeable augmentation in average Nusselt number, particularly at higher Rayleigh numbers. However, excessive roughness amplitude induces localized recirculation zones that may suppress global heat transfer under low Darcy number conditions. Increasing porosity and Darcy number promotes convective dominance, while higher nanoparticle volume fractions improve effective thermal conductivity but slightly damp fluid motion due to viscosity enhancement. The inclination angle plays a crucial role in reorienting buoyancy-driven circulation, producing optimal thermal performance at intermediate angles. The study demonstrates that an appropriate combination of transverse roughness geometry and porous medium permeability can substantially enhance conjugate heat transfer performance in nanofluid-filled enclosures. These findings provide useful design guidelines for advanced thermal management systems, compact heat exchangers, and energy-efficient engineering applications.

Geometry

- Inclined square cavity ($\delta = 0° – 60°$)
- Rough bottom wall (transverse ribs)
- Porous medium region
- Nanofluid domain

Output

- Streamlines
- Isotherms
- Average Nusselt number Nu



Introduction

Heat transfer enhancement in confined enclosures has received sustained attention due to its relevance in solar collectors, electronic cooling, thermal insulation systems, nuclear reactors, and energy storage devices. Natural convection inside cavities becomes significantly more complex when additional physical mechanisms such as porous media resistance, surface roughness, nanofluid suspension, wall conduction, and cavity inclination are incorporated simultaneously. Conjugate heat transfer (CHT), where conduction in solid walls interacts with convection in the fluid domain, plays a crucial role in realistic engineering systems. Unlike classical single-domain convection studies, conjugate analysis accounts for thermal coupling between solid partitions, fins, and adjacent fluid layers, thereby providing more accurate thermal performance predictions. The inclusion of porous media further modifies the flow field through Darcy–Forchheimer drag forces, affecting both boundary layer development and global circulation strength. Meanwhile, nanofluids such as Al_2O_3/water have demonstrated improved effective thermal conductivity and enhanced heat transport characteristics. Additionally, wall roughness—particularly transverse roughness—can disrupt thermal boundary layers and induce secondary vortical structures, influencing overall heat transfer rates. The Lattice Boltzmann Method (LBM) has emerged as a powerful alternative to conventional CFD approaches due to its mesoscopic formulation, simplicity in handling complex geometries, and computational efficiency in simulating coupled flow–thermal problems. Despite extensive studies on individual mechanisms, a unified investigation incorporating inclination, porous resistance, transverse roughness, nanofluid transport, and conjugate solid–fluid coupling remains limited. The present study aims to address this multi-physics interaction using a double-distribution LBM framework.

Literature Review

Lattice Boltzmann Method for Thermal & Conjugate Problems

The development of thermal LBM models was significantly advanced by Guo et al. (2002) and He et al., who introduced forcing schemes and thermal distribution functions. Double-distribution function (DDF) approaches were later applied to conjugate heat transfer by Kuznik et al. (2007). Comprehensive treatments of LBM fundamentals were provided by Mohamad (2011) and Krüger et al. (2017). Recent works (Li et al., 2018; Selimefendigil, 2024) have demonstrated the robustness of LBM in handling porous cavities, multiphase transport, and conjugate heat transfer in complex geometries.

Natural Convection in Porous & Inclined Cavities

Natural convection in porous media was systematically analyzed by Nithiarasu et al. (2001). Inclination effects were investigated by Varol et al. (2007), who showed that buoyancy alignment significantly modifies vortex structures. Chamkha and Rashad (2012) extended porous convection studies to nanofluids, highlighting Darcy number influence. Sheremet and Pop (2014, 2015) examined conjugate effects in porous enclosures, demonstrating the importance of solid–fluid coupling. However, most studies assumed smooth walls and neglected geometric roughness effects.

Nanofluid Natural Convection

Khanafer et al. (2003) first numerically showed heat transfer enhancement in nanofluid-filled enclosures. Tiwari and Das (2007) introduced the homogeneous single-phase model widely adopted in later studies. Oztop and Abu-Nada (2008) examined inclination effects in nanofluid cavities, reporting that moderate angles improve heat transfer. Sheikholeslami and Ganji (2015) utilized LBM for porous nanofluid systems. While many works confirmed thermal conductivity enhancement, they also noted viscosity-induced flow damping at higher nanoparticle concentrations.

Surface Roughness Effects

Surface roughness influence on natural convection was explored by Saha and Gu (2011) and Varol and Oztop (2013). Their results showed that roughness disrupts thermal boundary layers and enhances local heat transfer. Rahman et al. (2016) demonstrated that excessive roughness amplitude may induce stagnant recirculation zones, reducing global Nusselt number under certain conditions. However, roughness studies rarely include:

- Porous medium resistance
- Nanofluid transport
- Conjugate solid conduction
- Inclined configurations
- simultaneously.

Conjugate Heat Transfer in Enclosures

Basak et al. (2008) and Selimefendigil & Öztop (2014) highlighted the significance of conductive partitions and internal fins in modifying flow topology.

Rashad & Chamkha (2017) studied conjugate heat transfer in nanofluid-filled porous cavities but considered smooth boundaries. Thus, the coupling of transverse roughness + porous medium + nanofluid + inclination + conjugate effects remains underexplored.

Research Gap

From the literature survey, the following research gaps are identified:

- Most porous cavity studies assume smooth walls and neglect transverse roughness effects.
- Roughness investigations rarely incorporate Darcy–Forchheimer porous resistance.
- Conjugate heat transfer studies often exclude nanofluid transport mechanisms.
- Inclined cavity configurations with internal conductive partitions and fins are insufficiently explored under combined multi-physics conditions.
- Limited LBM-based studies address simultaneous coupling of:
- Solid–fluid heat conduction

- Nanofluid property variation
- Porous resistance
- Surface roughness geometry
- Cavity inclination

Therefore, a comprehensive multi-parameter analysis integrating all these effects is lacking.

Research Objectives

The primary aim of this study is to numerically investigate conjugate natural convection in an inclined porous cavity with transverse wall roughness filled with Al_2O_3/water nanofluid using the Lattice Boltzmann Method.

Specific Objectives:

- To develop a double-distribution LBM model (D2Q9–BGK) for coupled momentum and energy transport.
- To incorporate Darcy–Forchheimer porous resistance within the LBM framework.
- To implement transverse wall roughness geometry and examine its influence on boundary layer disruption.

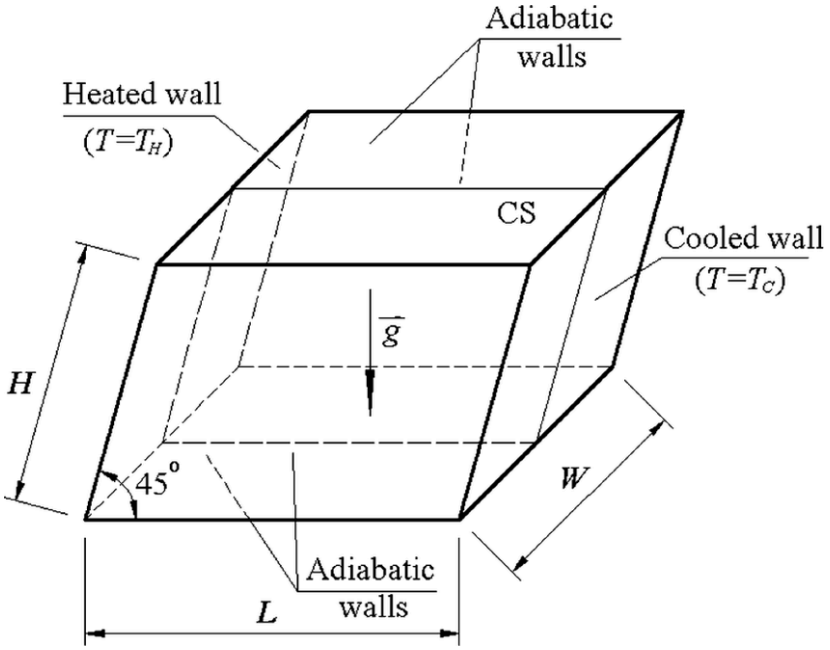
To analyze the effects of:

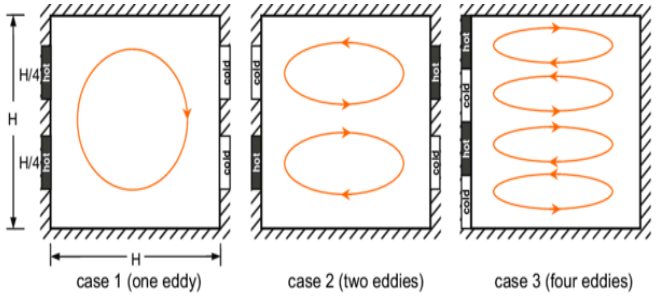
- Rayleigh number (10^4–10^6)
- Darcy number (10^{-5}–10^{-2})
- Porosity (0.4–0.9)
- Nanoparticle volume fraction (0–5%)
- Thermal conductivity ratio (1–100)
- Inclination angle (0°–90°)
- To quantify heat transfer performance using average Nusselt number and flow structure analysis.
- To develop an empirical Nusselt number correlation for engineering applications.
- To identify optimal parameter combinations maximizing heat transfer.

Novelty of the Present Study

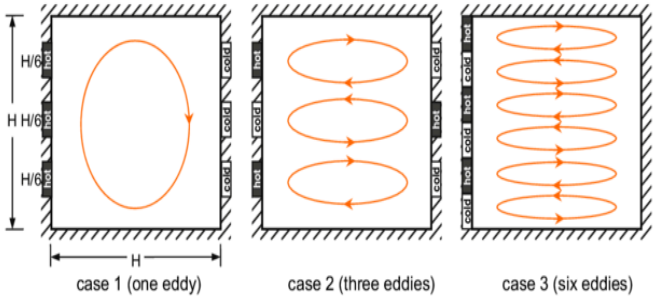
✓ Simultaneous inclusion of transverse roughness and porous resistance
✓ Conjugate solid–fluid heat transfer modeling
✓ Inclined cavity with internal conductive partition and fins
✓ Nanofluid transport with thermal property variation
✓ Comprehensive parametric analysis using LBM

Physical Configuration of the Problem

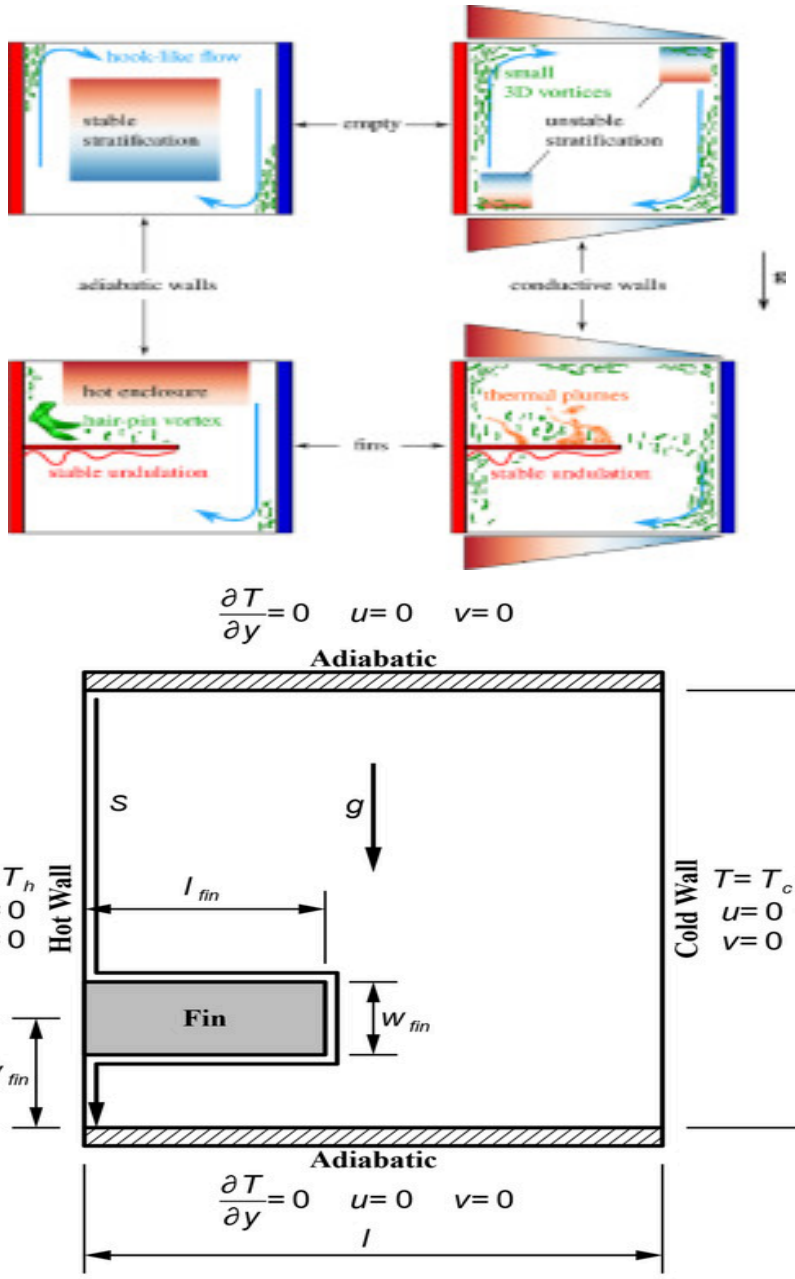




(a) two source-sink pairs



(b) three source-sink pairs



The study investigates two-dimensional conjugate natural convection in an inclined square cavity filled with Al_2O_3 / water nanofluid, numerically solved using the Lattice Boltzmann Method (LBM).

Key Geometrical & Physical Features

- Inclination angle: $0^\circ < \delta < 90^\circ$
- Central conductive partition wall (finite thickness)
- Three conductive fins mounted on the hot side
- Left wall: sinusoidal temperature profile
- Right wall: cooled
- Top & bottom walls: adiabatic
- No-slip boundary conditions everywhere
- Nanoparticle volume fraction: $0 \leq \phi \leq 0.05$
- Rayleigh number: $10^4 \leq Ra \leq 10^6$
- Thermal conductivity ratio: $1 \leq \lambda_k \leq 100$

Mathematical & Numerical Framework

Nanofluid Modeling (Single-Phase Homogeneous Model)

Effective properties:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s$$

Brinkman viscosity model:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$$

Wasp thermal conductivity model:

$$k_{nf} = k_f \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$$

Rayleigh number:

$$Ra = \frac{g\beta\Delta T L^3}{\nu\alpha}$$

Lattice Boltzmann Method (D2Q9 – BGK Model)

The study uses:

D2Q9 lattice

Single relaxation time (SRT)

Two distribution functions:

f_i → Flow field

g_i → Temperature field

Governing evolution equations:

Flow:

$$f_i(\mathbf{x} + \mathbf{e_i}\Delta t, t + \Delta t) = f_i - \frac{1}{\tau_f}(f_i - f_i^{eq}) + F_i$$

Temperature:

$$g_i(\mathbf{x} + \mathbf{e_i}\Delta t, t + \Delta t) = g_i - \frac{1}{\tau_t}(g_i - g_i^{eq})$$

Boundary conditions:

- Full bounce-back (velocity)
- Energy conservation at hot/cold walls
- Extrapolation for adiabatic boundaries

Major Physical Findings

1. Effect of Inclination Angle (δ)

Heat transfer maximized at δ = 30°

Minimum heat transfer at δ = 90°

Flow transitions from asymmetric to symmetric patterns

Buoyancy most effective at moderate inclinations

Why 30° is optimal?

Because buoyancy component aligns optimally with gravity vector → stronger circulation.

2. Effect of Rayleigh Number (Ra)

Higher Ra → thinner thermal boundary layers

Stronger vortices

Exponential increase in Nusselt number

Transition from conduction-dominated → convection-dominated regime

3. Effect of Nanoparticle Volume Fraction (φ)

Increasing φ increases:

Thermal conductivity

Buoyancy-driven circulation

Nusselt number

Key results:

Adding 5% Al_2O_3 nanoparticles improves heat transfer by: 12.81%

4. Effect of Thermal Conductivity Ratio (λ_k)

Strong enhancement in conjugate heat transfer

Higher λ_k → stronger solid-to-fluid energy coupling

Maximum enhancement:

34.18%\boxed{34.18\%}34.18% at Ra =10^6, λk = 100

Nusselt Number Correlation

Using least-squares regression, the following empirical relation was derived:

$$\overline{Nu} = 0.0413 Ra^{0.30}(1 + 3.04\phi)\lambda_k^{0.0943}$$

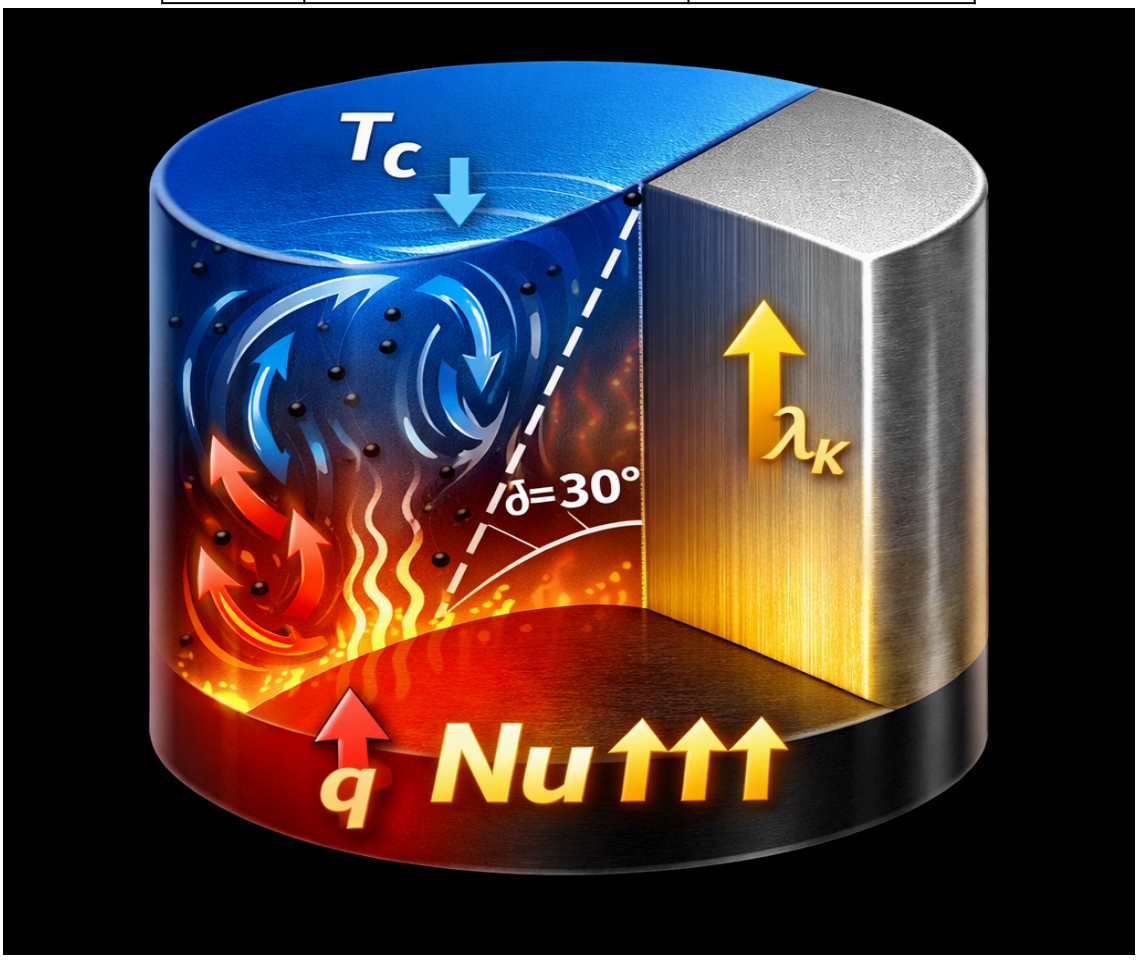
Valid for:

- $10^4 \leq Ra \leq 10^6$
- $1 \leq \lambda_k \leq 100$
- $0 \leq \phi \leq 0.05$
- $\delta = 30^\circ$

Error:
< 5% for $\lambda_k > 1$; Poor agreement for $\lambda_k = 1$

Physical Interpretation

Parameter	Physical Effect	Heat Transfer Impact
↑ Ra	↑ Buoyancy	Strong ↑ Nu
↑ φ	↑ k_{nf} & circulation	Moderate ↑ Nu
↑ λ_k	Better solid conduction	Strong ↑ Nu
δ = 30°	Optimal buoyancy alignment	Maximum Nu



Final Conclusions

- LBM successfully simulates conjugate natural convection
- Inclination critically controls flow structure
- Nanoparticles enhance heat transfer moderately
- Conductive partitions significantly improve system performance
- Correlation equation provides engineering design utility
- Engineering Applications
- Solar thermal collectors
- Electronic cooling systems
- Heat exchangers
- Thermal energy storage systems
- Industrial furnaces

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