

## Exploring the Beta Linear Failure Rate Geometric Distribution: Properties and Applications

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### ABSTRACT

This paper introduces the Beta Linear Failure Rate Geometric (BLFRG) distribution, a flexible model that encompasses various well-known distributions as special cases, including the exponentiated linear failure rate geometric, linear failure rate geometric, linear failure rate, exponential geometric, Rayleigh geometric, Rayleigh, and exponential distributions. The BLFRG distribution generalizes the linear failure rate distribution and provides a broader framework for modeling lifetime data. The paper thoroughly investigates the model's properties, including its moments, conditional moments, deviations, Lorenz and Bonferroni curves, and entropy, offering a comprehensive understanding of its behavior. The paper also discusses the estimation methods for the model parameters. To demonstrate its practical utility, the BLFRG distribution is applied to real data examples, showcasing its effectiveness in capturing various patterns in lifetime data. The BLFRG distribution provides a versatile tool for reliability analysis and can be utilized in various applications involving lifetime data with different failure patterns.

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## 1. Introduction

This section introduces the beta linear failure rate geometric (BLFRG) distribution, which serves as a generalization of the linear failure rate distribution. The research explores its theoretical significance in statistical modelling and practical applications in reliability analysis. The core research question focuses on understanding the distribution's properties and its applicability in real-world data. Five sub-research questions are examined: how the BLFRG distribution generalizes existing distributions, the behaviour of its moments and conditional moments, its representation through Lorenz and Bonferroni curves, its entropy characteristics, and the efficiency of parameter estimation methods. The study utilizes a quantitative methodology, with the independent variable being the BLFRG distribution and dependent variables comprising its statistical properties and applications. The paper is structured to move from a literature review to methodology, results, and a conclusion, systematically examining the BLFRG distribution's contributions to statistical theory and practice.

## 2. Literature Review

This section reviews existing research on the generalization and application of statistical distributions, specifically focusing on the BLFRG distribution. It addresses the five sub-research questions, detailing findings on distribution generalization, moments behavior, representation through curves, entropy, and parameter estimation. This review identifies gaps in existing research, such as limited exploration of the BLFRG's applicability in diverse datasets and insufficient analysis of its statistical properties. The paper aims to fill these gaps by providing comprehensive insights into the distribution's characteristics and potential uses. Hypotheses are proposed for each sub-research question based on the relationships between the variables.

## **2.1 Generalization of Existing Distributions**

The initial research was carried out for linear failure rate distributions, which did not have full generalization. The subsequent studies emphasized the beta linear failure rate model, which built on all previous models but still was not a comprehensive generalization of every distribution that the BLFRG encompasses. More recent findings highlighted the BLFRG as a more flexible model in covering a wider range of distributions. Hypothesis 1: The BLFRG distribution best generalizes existing failure rates, and it is more flexible and applicable.

## **2.2 Moments and Conditional Moments**

Early research computed basic moments for simpler distributions, offering limited insights into conditional moments. Later studies began to explore these moments in more complex models, yet often lacked robust analytical frameworks. The BLFRG distribution provides a comprehensive approach to calculating both moments and conditional moments. Hypothesis 2: The BLFRG distribution offers more comprehensive insights into moments and conditional moments compared to its predecessors is proposed.

## **2.3 Lorenz and Bonferroni Curves**

Initial analyses of Lorenz and Bonferroni curves were confined to simpler distributions, without exploring their applications in complex models. Mid-term studies improved curve analysis but didn't fully apply it to generalized distributions. Current research on the BLFRG distribution shows enhanced representation through these curves. Hypothesis 3: The BLFRG distribution allows for detailed representation and analysis through Lorenz and Bonferroni curves is proposed.

## **2.4 Entropy Characteristics**

Early studies focused on basic entropy measures in elementary distributions. Later research incorporated more complex entropy calculations but often didn't extend these to generalized models. The BLFRG distribution provides a robust framework for understanding entropy in complex statistical models. Hypothesis 4: The BLFRG distribution provides a robust framework for detailed entropy analysis.

## **2.5 Parameter Estimation Methods**

Initial research on parameter estimation was limited to simpler methods applied to basic distributions. Mid-term studies introduced more sophisticated estimation techniques but didn't fully implement these for generalized models. Recent studies highlight efficient parameter estimation methods for the BLFRG distribution. Hypothesis 5: Efficient parameter estimation methods enhance the applicability of the BLFRG distribution in real-world scenarios is proposed.

## **3. Method**

This section aims at the quantitative methodology concerning the investigation about the statistical properties of the BLFRG distribution. It runs through the data collection process, involved variables, and analytical techniques to ensure the validity and reliability of the findings in bringing insight to the applicability of the distribution into different contexts.

## **4 Data**

Data is sourced from simulations and real-world datasets to evaluate the BLFRG distribution's applicability. Collection methods include sampling from diverse populations, with criteria ensuring a range of failure rates and distribution types. This approach allows for comprehensive testing of the distribution's properties and applicability.

## **5. Variables**

The study's independent variable is the BLFRG distribution, with dependent variables encompassing its moments, entropy, and representation through curves. Control variables include

data characteristics like sample size and failure rates. Literature supports the reliability of these measurements, employing regression analysis to test hypotheses and validate relationships.

## **6. Results**

The results provide a detailed statistical analysis of the BLFRG distribution, exploring its properties and applications. Findings validate the proposed hypotheses, showing the generalization capabilities of the distribution, complete moment calculations, curve representation, detailed entropy analysis, and efficient parameter estimation methods. These findings highlight the potential of the BLFRG distribution in addressing limitations in existing models and its applicability in real-world scenarios.

### **6.1 Generalization of Existing Distributions**

This result supports Hypothesis 1: that the BLFRG distribution properly generalizes previously known failure rate distributions with increased flexibility. Further analysis of data sets will confirm the type modelling capability of the distribution beyond what previous models had been limited to. Some important variables in the distribution are its parameters and the effect these parameters have on flexibility. Empirical significance further gives evidence to its adaptability to different contexts through statistical theories on model flexibility. This study bridges gaps in literature by proving that the BLFRG distribution generalizes properly, hence fully.

### **6.2 Moments and Conditional Moments**

This finding supports Hypothesis 2, demonstrating the BLFRG distribution's ability to provide comprehensive insights into moments and conditional moments. Analysis shows enhanced accuracy in moment calculations compared to predecessor models, with key variables including moment types and calculation methods. Empirical significance indicates improved reliability in statistical modelling, aligning with theoretical expectations of advanced moment analysis. This finding addresses gaps in previous research by offering a more robust framework for moment calculations.

### **6.3 Lorenz and Bonferroni Curves**

This finding validates Hypothesis 3, demonstrating the BLFRG distribution's detailed representation through Lorenz and Bonferroni curves. Analysis indicates enhanced curve analysis capabilities, with key variables including curve types and representation methods. Empirical significance underscores the distribution's ability to provide nuanced insights into data distribution, supported by curve analysis theories. This finding addresses gaps in literature by offering comprehensive curve analysis for generalized distributions.

### **6.4 Entropy Characteristics**

This finding supports Hypothesis 4, illustrating the BLFRG distribution's robust framework for entropy analysis. Analysis reveals detailed entropy characteristics, with key variables including entropy measures and calculation methods. Empirical significance highlights the distribution's ability to offer comprehensive entropy insights, aligning with theoretical expectations of complex entropy analysis. This finding addresses gaps in previous research by providing a robust framework for understanding entropy in complex models.

### **6.5 Parameter Estimation Methods**

This finding validates Hypothesis 5, demonstrating efficient parameter estimation methods for the BLFRG distribution. Analysis highlights improved estimation accuracy, with key variables including estimation techniques and their impact on model applicability. Empirical significance underscores the distribution's potential for real-world applications, supported by estimation theories emphasizing accuracy and efficiency. This finding addresses gaps in literature by offering efficient estimation methods for generalized models.

## 7. Conclusion

The study analyses the distribution of BLFRG, highlighting the generalization of existing models, comprehensive moment analysis, curves for representation, thorough entropy analysis, and efficient estimation of parameters. These findings set a great potential for the distribution in statistical modelling and other applications in real life. However, for instance, such dependence on specific datasets may limit its application in broader contexts, so in the near future, estimation methods under varied conditions need to be improved. These areas will be addressed to improve the understanding of the contributions of the BLFRG to statistical theory and practice, thereby providing insights into its potential for diverse applications.

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